

CALCULUS

This Summer Review Packet is due the FIRST day of school, ~~September 8, 2005~~. I expect you to have it NEATLY COMPLETED and WITH YOU on the first day of school. There may be one or two problems you have difficulty with, circle those so we can play with them on the first day. My class expectations, etc. will be posted online, so I will spend no more than 10 min. on that, and then we will start working.

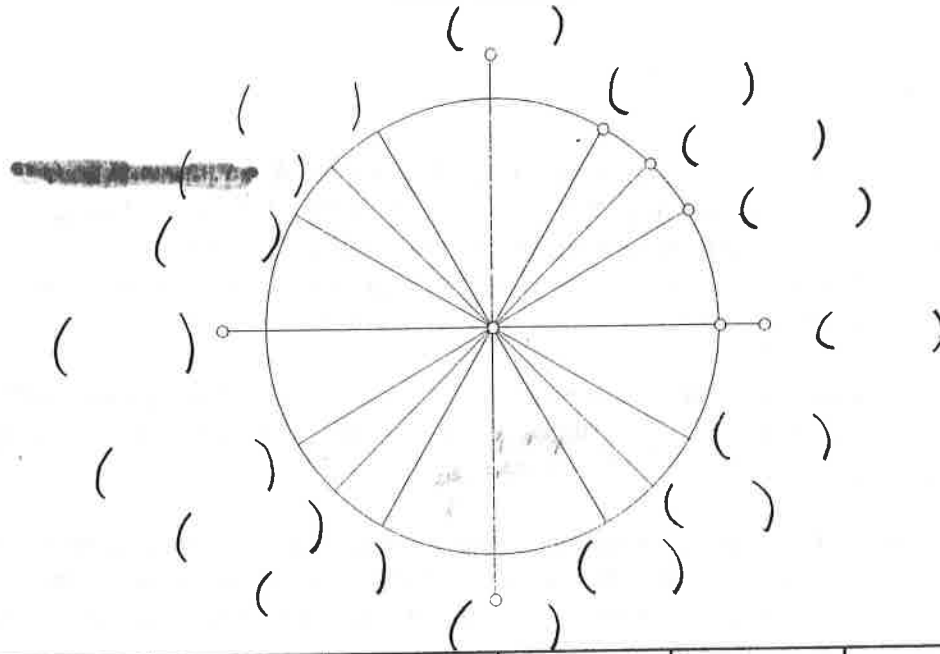
The Chart worksheet is a quiz you must correctly, completely fill in, in 5 min. or less. Each time you take the quiz, you will get -2. For example, if it takes 4 tries to complete then you get 42/50.

Please do all graphing calculator sketches neatly and large enough to read the scale. *No scale = no credit* for all graphs. One of the main ideas behind the graphing exercises is to become absolutely fluent in many of the uses of your calculator. You will use the (2nd) CALC and MATH menus often.

You also need to read the book A Tour of the Calculus by David Berlinski. There will be a couple other reading assignments throughout the year.

My website will be updated in preparation for fall by July 15:
www.olypen.com/jschmitt

THE CHART



m^R	m^θ	$\cos \theta$	$\sin \theta$	$\tan \theta$	$\sec \theta$	$\csc \theta$	$\cot \theta$
0							
$\pi/6$							
$\pi/4$							
$\pi/3$							
$\pi/2$							
$2\pi/3$							
$3\pi/4$							
$5\pi/6$							
π							
$7\pi/6$							
$5\pi/4$							
$4\pi/3$							
$3\pi/2$							
$5\pi/3$							
$7\pi/4$							
$11\pi/6$							

AP CALCULUS SUMMER WORKSHEET

DUE: First day of school.

This assignment is to be done at your leisure during the summer. It is designed to help you become comfortable with your graphing calculator. You will need to read the manual to understand how your calculator works. It is important that you gain these skills over the summer so that we can spend our time talking about calculus rather than how to use the calculator.

Graph the parent function of each set using your calculator. Draw a quick sketch on your paper of each additional equation in the family. Check your sketch with the graphing calculator. Be sure to label the scale of each graph.

1) Parent Function: $y = x^2$

a) $y = x^2 - 5$

b) $y = x^2 + 3$

c) $y = (x-10)^2$

d) $y = (x+8)^2$

e) $y = 4x^2$

f) $y = 0.25x^2$

g) $y = -x^2$

h) $y = -(x+3)^2 + 6$

i) $y = (x+4)^2 - 8$

j) $y = -2(x+1)^2 + 4$

k) $y = \frac{1}{3}(x-6)^2 - 6$

l) $y = -3(x+2)^2 - 2$

2) Parent Function: $y = \sin(x)$ (set mode to RADIANS)

a) $y = \sin(2x)$

b) $y = \sin(x) - 2$

c) $y = 2 \sin(x)$

d) $y = 2\sin(2x) + 2$

3) Parent Function: $y = \cos(x)$

a) $y = \cos(3x)$

b) $y = \cos(x/2)$

c) $y = 2\cos(x) + 2$

d) $y = -2\cos(x) - 1$

4) Parent Function: $y = x^3$

a) $y = x^3 + 2$

b) $y = -x^3$

b) $y = x^3 - 5$

c) $y = -x^3 + 3$

e) $y = (x-4)^3$

f) $y = (x-1)^3 - 4$

g) $y = -2(x+2)^3 + 1$

h) $y = x^3 + x$

5) Parent Function: $y = \sqrt{x}$

a) $y = \sqrt{x} - 2$

b) $y = \sqrt{-x}$

c) $y = \sqrt{x} + 5$

d) $y = \sqrt{6 - x}$

e) $y = -\sqrt{x}$

f) $y = -\sqrt{-x}$

g) $y = \sqrt{x + 2}$

h) $y = \sqrt{2x - 6}$

i) $y = -2\sqrt{x}$

j) $y = -\sqrt{4 - x}$

6) Parent Function: $y = \ln(x)$

a) $y = \ln(x+3)$

b) $y = \ln(x) + 3$

c) $y = \ln(x-2)$

d) $y = \ln(-x)$

e) $y = -\ln(x)$

f) $y = \ln(|x|)$

g) $y = \ln(2x) - 4$

h) $y = -3\ln(x) + 1$

7) Parent Function: $y = e^x$

a) $y = e^{2x}$

b) $y = e^{x-2}$

c) $y = e^{2-x}$

d) $y = e^{2x} + 3$

e) $y = -e^x$

f) $y = e^{-x}$

g) $y = 2 - e^x$

h) $y = e^{0.5x}$

8) Parent Function $y = a^x$

a) $y = 5^x$

b) $y = 2^x$

c) $y = 3^{-x}$

d) $y = \frac{1}{2}^x$

e) $y = 4^{x-3}$

f) $y = 2^{x-3} + 2$

9) Parent Function: $y = 1/x$

a) $y = 1/(x-2)$

b) $y = -1/x$

c) $y = 1/(x+4)$

d) $y = 2/(5-x)$

10) Parent Function: $y = [x]$

Note: $[x]$ is the ^{greatest integer} Integer Part of x . It is found in the MATH menu, NUM submenu. $\text{int}(\)$

a) $y = [x] + 2$

b) $y = [x-3]$

c) $y = [3x]$

d) $y = [0.25x]$

e) $y = 3 - [x]$

e) $y = 2[x] - 1$

11) Resize your viewing window to $[0,1] \times [0,1]$. Graph all of the following functions in the same window. List the functions from the highest graph to the lowest graph. How do they compare for values of $x > 1$?

a) $y = x^2$

b) $y = x^3$

c) $y = \sqrt{x}$

d) $y = x^{2/3}$

e) $y = |x|$

f) $y = x^4$

12) Given: $f(x) = x^4 - 3x^3 + 2x^2 - 7x - 11$
Find all roots to the nearest 0.001

13) Given: $f(x) = 3 \sin 2x - 4x + 1$ from $[-2\pi, 2\pi]$
Find all roots to the nearest 0.001.
Note: All trig functions are done in radian mode.

14) Given: $f(x) = 0.7x^2 + 3.2x + 1.5$

Find all roots to the nearest 0.001.

15) Given: $f(x) = x^4 - 8x^2 + 5$
Find all roots to the nearest 0.001.

16) Given: $f(x) = x^3 + 3x^2 - 10x - 1$
Find all roots to the nearest 0.001

17) Given: $f(x) = 100x^3 - 203x^2 + 103x - 1$
Find all roots to the nearest 0.001

18) Given: $f(x) = |x-3| + |x| - 6$
Find all roots to the nearest 0.001

19) Given: $f(x) = |x| - |x-6| = 0$
Find all roots to the nearest 0.001

Solve the following inequalities

20) $x^2 - x - 6 > 0$

21) $x^2 - 2x - 5 \geq 3$

22) $x^3 - 4x < 0$

For each of the following (problems 23-26)

- a) Sketch the graph of $f(x)$
- b) Sketch the graph of $|f(x)|$
- c) Sketch the graph of $f(|x|)$
- d) Sketch the graph of $f(2x)$
- e) Sketch the graph of $2f(x)$

23) $f(x) = 2x+3$

24) $f(x) = x^2 - 5x - 3$

25) $f(x) = 2\sin(3x)$

26) $f(x) = -x^3 - 2x^2 + 3x - 4$

27) Let $f(x) = \sin x$
Let $g(x) = \cos x$

- a) Sketch the graph of f^2
- b) Sketch the graph of g^2
- c) Sketch the graph of $f^2 + g^2$

- 28) Given: $f(x) = 3x+2$
 $g(x) = -4x-2$
Find the point of intersection
- 29) Given: $f(x) = x^2 - 5x + 2$
 $g(x) = 3-2x$
Find the coordinates of any points of intersection.
- 30) How many times does the graph of $y = 0.1x$ intersect the graph of $y = \sin(2x)$?
- 31) Given: $f(x) = x^4 - 7x^3 + 6x^2 + 8x + 9$
- Determine the x- and y-coordinates of the lowest point on the graph.
 - Size the x-window from $[-10,10]$. Find the highest and lowest values of $f(x)$ over the interval $-10 \leq x \leq 10$
 - Find the highest and lowest values of $f(x)$ if $-2 \leq x \leq 6$.
 - Suppose the *range* is from 10 to 30 in Quadrant II. Determine the x-coordinates (domain) of the function that will cover the range.

AP Calculus Summer Review Packet

This packet is a review of the entering objectives for AP Calculus and is due on the **first day back to school**. It is to be done NEATLY and on a SEPARATE sheet of paper. Have a great summer.

I. Simplify. Show the work that leads to your answer.

1. $\frac{x-4}{x^2-3x-4}$

2. $\frac{x^3-8}{x-2}$

3. $\frac{5-x}{x^2-25}$

4. $\frac{x^2-4x-32}{x^2-16}$

II. Trigonometric Identities.

1. Pythagorean = _____

2. $\cos 2x =$ _____

3. $\sin 2x =$ _____

III. Simplify each expression.

1. $\frac{1}{x+h} - \frac{1}{x}$

2. $\frac{\frac{2}{x^2}}{\frac{10}{x^5}}$

3. $\frac{\frac{1}{3+x} - \frac{1}{3}}{x}$

4. $\frac{2x}{x^2-6x+9} - \frac{1}{x+1} - \frac{8}{x^2-2x-3}$

IV. Solve for z:

1. $4x + 10yz = 0$

2. $y^2 + 3yz - 8z - 4x = 0$

V. If: $f(x) = \{(3,5), (2,4), (1,7)\}$ $g(x) = \sqrt{x-3}$ $h(x) = \{(3,2), (4,3), (1,6)\}$ $k(x) = x^2 + 5$
determine each of the following:

1. $(f+h)(1) =$

2. $(k-g)(5) =$

3. $(f \circ h)(3) =$

4. $(g \circ k)(7) =$

5. $f^{-1}(x) =$

6. $k^{-1}(x) =$

7. $\frac{1}{f(x)} =$

8. $(kg)(x) =$

VI. Miscellaneous: Follow the directions for each problem.

1. Evaluate $\frac{f(x+h)-f(x)}{h}$ and simplify if $f(x) = x^2 - 2x$.

2. Expand $(x+y)^3$

3. Simplify: $x^{\frac{3}{2}}(x + x^{\frac{5}{2}} - x^2)$

4. Eliminate the parameter and write a rectangular equation for $x = t^2 + 3$
 $y = 2t$

AP Calculus Summer Review Packet

VII. Expand and simplify

1. $\sum_{n=0}^4 \frac{n^2}{2}$

2. $\sum_{n=1}^3 \frac{1}{n^3}$

VIII. Simplify

1. $\frac{\sqrt{x}}{x}$

2. $e^{\ln 3}$

3. $e^{(1+\ln x)}$

4. $\ln 1$

5. $\ln e^7$

6. $\log_3(1/3)$

7. $\log_{1/2} 8$

8. $\ln \frac{1}{2}$

9. $e^{3 \ln x}$

10. $\frac{4xy^{-2}}{12x^{\frac{1}{3}}y^{-5}}$

11. $27^{2/3}$

12. $(5a^{2/3})(4a^{3/2})$

13. $(4a^{5/3})^{3/2}$

14. $\frac{3(n+1)!}{5n!}$

IX. Using the point-slope form $y - y_1 = m(x - x_1)$, write an equation for the line

1. with slope -2 , containing the point $(3, 4)$

1. _____

2. containing the points $(1, -3)$ and $(-5, 2)$

2. _____

3. with slope 0 , containing the point $(4, 2)$

3. _____

4. parallel to $2x - 3y = 7$ and passes through $(5, 1)$

4. _____

5. perpendicular to the line in problem #1, containing the point $(3, 4)$

5. _____

X. Given the vectors $\mathbf{v} = -2\mathbf{i} + 5\mathbf{j}$ and $\mathbf{w} = 3\mathbf{i} + 4\mathbf{j}$, determine

1. $\frac{1}{2}\mathbf{v}$

2. $\mathbf{w} - \mathbf{v}$

3. length of \mathbf{w}

4. the unit vector for \mathbf{v}

XI. Without a calculator, determine the exact value of each expression.

1. $\sin 0$

2. $\sin \frac{\pi}{2}$

3. $\sin \frac{3\pi}{4}$

4. $\cos \pi$

5. $\cos \frac{7\pi}{6}$

6. $\cos \frac{\pi}{3}$

7. $\tan \frac{7\pi}{4}$

8. $\tan \frac{\pi}{6}$

9. $\tan \frac{2\pi}{3}$

10. $\tan \frac{\pi}{2}$

11. $\cos(\sin^{-1} \frac{1}{2})$

12. $\sin^{-1}(\sin \frac{7\pi}{6})$

XII. For each function, determine its domain and range.

1. $y = \sqrt{x-4}$

2. $y = \sqrt{x^2-4}$

3. $y = \sqrt{4-x^2}$

4. $y = \sqrt{x^2+4}$

AP Calculus Summer Review Packet

XIII. Determine all points of intersection.

1. parabola $y = x^2 + 3x - 4$ and
line $y = 5x + 11$

2. $y = \cos x$ and $y = \sin x$ in the
first quadrant

XIV. Solve for x , where x is a real number. Show the work that leads to your solution.

- | | | |
|---|---|---------------------------|
| 1. $x^2 + 3x - 4 = 14$ | 2. $\frac{x^4 - 1}{x^3} = 0$ | 3. $(x - 5)^2 = 9$ |
| 4. $2x^2 + 5x = 8$ | 5. $(x + 3)(x - 3) > 0$ | 6. $x^2 - 2x - 15 \leq 0$ |
| 7. $12x^2 = 3x$ | 8. $\sin 2x = \sin x, 0 \leq x \leq 2\pi$ | 9. $ x - 3 < 7$ |
| 10. $(x + 1)^2(x - 2) + (x + 1)(x - 2)^2 = 0$ | | 11. $27^{2x} = 9^{x-3}$ |
| 12. $\log x + \log(x - 3) = 1$ | 13. $e^{3k} = 5$ | 14. $\ln y = 2t - 3$ |
-

XV. Graph each function. Give its domain and range.

- | | | |
|------------------------------|--|--------------------------|
| 1. $y = \sin x$ | 2. $y = \cos x$ | 3. $y = \tan x$ |
| 4. $y = x^3 - 2x^2 - 3x$ | 5. $y = x^2 - 6x + 1$ | 6. $y = \frac{x+4}{x-1}$ |
| 7. $y = \frac{x^2 - 4}{x+2}$ | 8. $y = e^x$ | 9. $y = \sqrt{x}$ |
| 10. $y = \sqrt[3]{x}$ | 11. $y = \ln x$ | 12. $y = x + 3 - 2$ |
| 13. $y = \frac{1}{x}$ | 14. $y = \begin{cases} x^2 & \text{if } x < 0 \\ x + 2 & \text{if } 0 \leq x \leq 3 \\ 4 & \text{if } x > 3 \end{cases}$ | |

